

ELIMINATION OF SPURIOUS STATES OF OSCILLATOR SHELL MODEL

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Abstract

A new method for elimination of the 'spurious' states comprising requirement of translational invariance and simplicity of the enumeration scheme of antisymmetric A -particle states has been developed. The method presented enables one to project off the space of purely spurious antisymmetric A -particle oscillator functions with singled out dependence on intrinsic coordinates of two last particles onto the subspace of the nonspurious motion. The method is based on constructing the matrix elements of the centre-of-mass Hamiltonian and spectral decomposition of the corresponding real symmetric matrix. The obtained coefficients of the expansion of the antisymmetric A -particle oscillator functions in terms of the ones with singled out dependence on intrinsic coordinates of two last particles and with eliminated spurious states are required for calculation of two-particle oscillator intrinsic density matrices. Software tools based on a root rational fraction expression of numbers for the theoretical formulation presented have been developed.

1 Introduction

The translationally invariant shell model, originally proposed by Kretzschmar [1, 2], offers a very sophisticated method for characterization of antisymmetric states and producing the intrinsic coefficients of fractional parentage (ICFP's). A description of the many-particle wave functions in terms of relative coordinates turns out to be unfeasible because of the antisymmetrization requirements of the Pauli principle. As a consequence the ICFP's are available only in the E_{\min} approach and for the excited states with the maximum orbital momentum [3, 4].

More promising is to establish and use the relation of the translationally invariant shell model (for an arbitrary excitation) and the ordinary oscillator shell model. The

relation of this type makes it possible to obtain the results that obey the principle of translational invariancy by means of calculations within the framework of considerably simpler oscillator shell model.

However, the ordinary oscillator shell model is inherent with the so-called 'spurious' states corresponding to the centre-of-mass motion of the nucleus [5]. The ordinary oscillator shell model Hamiltonian is not invariant with respect to translations. Hence, the wave functions under calculation may contain nonphysical components, which are called spurious. The latter describe excitations of the centre-of-mass motion of the nucleus that are introduced with the single-particle potentials in the laboratory system. These excitations should be eliminated, since only the intrinsic excitations corresponds to the genuine excitations of the physical system that are observed.

The description of intrinsic properties of atomic nuclei may be simplified using the two-particle intrinsic density matrices instead of wave functions [6]. However, according the procedure proposed in Ref. [6], for calculation of the two-particle intrinsic density matrix for an arbitrary number of oscillator quanta, the expansion coefficients for the product of the centre-of-mass ground state function and the intrinsic wave function in terms of antisymmetric A -particle functions are needed.

In the present work we show that the coefficients of this type may be obtained by generating purely spurious antisymmetric A -particle oscillator functions with singled out dependence on intrinsic coordinates of two last particles and, then, by projecting them off onto the subspace of the nonspurious motion. This may be accomplished by diagonalizing the centre-of-mass Hamiltonian in the basis set of antisymmetric A -particle oscillator functions with singled out dependence on intrinsic coordinates of two last particles and choosing the subspace of its eigenvectors corresponding to the minimal eigenvalue equal to $\frac{3}{2}$. This procedure can be successful only when the non-truncated configuration space is taken into account. It implies that a large number of antisymmetric A -particle states has to be used. Such large scale calculations can be carried out by using the algorithms presented in Ref. [6, 7] for enumeration and formation of antisymmetric A -particle states. The proposed method for elimination of

'spurious' states is the first one which enables practically production of two-particle oscillator intrinsic density matrices for an arbitrary excitation.

2 The centre-of-mass Hamiltonian matrix

The functions from the subspace of the nonspurious motion are the eigenfunctions of the centre-of-mass Hamiltonian. Thus, projection onto the subspace of the nonspurious motion implies such superposition of the oscillator shell model functions which diagonalize the centre-of-mass Hamiltonian matrix. So, at first the matrix elements of the centre-of-mass Hamiltonian between the oscillator shell model functions (in the configuration space) should be obtained, at second, the eigenvalue problem should be solved, and at last the submatrix corresponding to the zero excitation energy should be extracted.

The Hamiltonian of the many-particle oscillator shell model may be divided in two parts: the centre-of-mass Hamiltonian $H_{\text{c.m.}}$ and the intrinsic Hamiltonian $H_{\text{intr.}}$

$$H = H_{\text{c.m.}} + H_{\text{intr.}} \quad (1)$$

The centre-of-mass Hamiltonian $H_{\text{c.m.}}$ describes the motion of the centre of mass of the A -particle system and is of the form

$$H_{\text{c.m.}} = \frac{\hbar\omega}{2} \left\{ -\frac{b^2}{A} \vec{\nabla}_R^2 + \frac{A}{b^2} R^2 \right\} \quad (2)$$

Here $\vec{R} = \frac{1}{A} \sum_{i=1}^A \vec{r}_i$ denotes the centre-of-mass position vector, $\vec{\nabla}_R = \sum_{i=1}^A \vec{\nabla}_i$, and ω is related with the usual oscillator length via $b = \sqrt{\hbar/m\omega}$, where m denotes the nucleon mass. In the one-particle coordinates the Hamiltonian (2) can be written as

$$H_{\text{c.m.}} = \frac{\hbar\omega}{2A} \left\{ -b^2 \left(\sum_{i=1}^A \vec{\nabla}_i \right)^2 + \frac{1}{b^2} \left(\sum_{i=1}^A \vec{r}_i \right)^2 \right\} \quad (3)$$

By using b as our unit of length and rewriting Eq. (3) in terms of dimensionless quantities $\vec{y}_{(i)} = \vec{r}_i/b$ and $\vec{\nabla}_{(i)} = b\vec{\nabla}_i$, one obtains

$$H_{\text{c.m.}} = \frac{\hbar\omega}{2A} \sum_{i,j=1}^A \left\{ -\vec{\nabla}_{(i)} \vec{\nabla}_{(j)} + \vec{y}_{(i)} \vec{y}_{(j)} \right\} . \quad (4)$$

In order to separate mixed terms one can use the identity valid for an arbitrary entries a_{ij} symmetrical with respect to the index permutation $a_{ij} = a_{ji}$

$$\sum_{i,j=1}^A a_{ij} = \frac{1}{A-1} \sum_{\substack{i,j=1 \\ i < j}}^A [a_{ii} + 2(A-1)a_{ij} + a_{jj}] . \quad (5)$$

This leads to representation of $H_{\text{c.m.}}$ in terms of single-particle and two-particle operators

$$\begin{aligned} H_{\text{c.m.}} = & \frac{\hbar\omega}{2} \frac{2}{A(A-1)} \sum_{\substack{i,j=1 \\ i < j}}^A \left\{ A \frac{1}{2} \left[-\vec{\nabla}_{(i)}^2 + \vec{y}_{(i)}^2 \right] + A \frac{1}{2} \left[-\vec{\nabla}_{(j)}^2 + \vec{y}_{(j)}^2 \right] \right. \\ & \left. - (A-1) \frac{1}{2} \left[-(\vec{\nabla}_{(i)} - \vec{\nabla}_{(j)})^2 + (\vec{y}_{(i)} - \vec{y}_{(j)})^2 \right] \right\} . \end{aligned} \quad (6)$$

Now, the matrix element of the centre-of-mass Hamiltonian between the oscillator shell model functions assumes the form

$$\begin{aligned} & \langle EK \Delta J \Pi T | \frac{H_{\text{c.m.}}}{\hbar\omega} | EK' \Delta' J \Pi T \rangle \\ &= \frac{1}{2} \sum_{\substack{\overline{(EK \Delta J \Pi T)} \\ J'' T''}} \sum_{\substack{(elj)_{A-1}, (elj)_A \\ (elj)'_{A-1}, (elj)'_A}} \langle \overline{(EK \Delta J \Pi T)}; ((elj)_{A-1}, (elj)_A) J'' T'' | | EK \Delta J \Pi T \rangle \\ & \times \langle \overline{(EK \Delta J \Pi T)}; ((elj)'_{A-1}, (elj)'_A) J'' T'' | | EK' \Delta' J \Pi T \rangle \\ & \times \langle ((elj)_{A-1}, (elj)_A) J'' T'' | \left\{ A \frac{1}{2} \left[-\vec{\nabla}_{(A-1)}^2 + \vec{y}_{(A-1)}^2 \right] + A \frac{1}{2} \left[-\vec{\nabla}_{(A)}^2 + \vec{y}_{(A)}^2 \right] \right. \\ & \left. - (A-1) \frac{1}{2} \left[-(\vec{\nabla}_{(A-1)} - \vec{\nabla}_{(A)})^2 + (\vec{y}_{(A-1)} - \vec{y}_{(A)})^2 \right] \right\} | ((elj)'_{A-1}, (elj)'_A) J'' T'' \rangle . \end{aligned} \quad (7)$$

Here the usual shell model fractional parentage expansion is used to separate out two last particles in the list, and the notation follows that introduced in Ref. [6]. The terms corresponding to the harmonic oscillator operators of one-particle type may be easily obtained by means of the eigenvalue equation

$$\frac{1}{2} \left(-\vec{\nabla}^2 + \vec{\zeta}^2 \right) \phi_{\vec{\epsilon}}(\vec{\zeta}) = \left(\tilde{\epsilon} + \frac{3}{2} \right) \phi_{\vec{\epsilon}}(\vec{\zeta}) . \quad (8)$$

Here $\phi_{\tilde{e}}(\vec{\zeta})$ is the single-particle harmonic oscillator function in argument $\vec{\zeta}$ with oscillator quantum number \tilde{e} . So, since many-particle oscillator shell model functions can be expressed in terms of single-particle harmonic oscillator functions, the following equality is rather straightforward

$$\begin{aligned}
& \sum_{\substack{\overline{(EK\Delta J\bar{I}\bar{I}T)}}_{J''T''}} \sum_{\substack{(elj)_{A-1}, (elj)_A \\ (elj)'_{A-1}, (elj)'_A}} \langle \overline{(EK\Delta J\bar{I}\bar{I}T)}; ((elj)_{A-1}, (elj)_A) J''T'' || EK\Delta J\bar{I}\bar{I}T \rangle \\
& \times \langle \overline{(EK\Delta J\bar{I}\bar{I}T)}; ((elj)'_{A-1}, (elj)'_A) J''T'' || EK'\Delta' J\bar{I}\bar{I}T \rangle \\
& \times \langle ((elj)_{A-1}, (elj)_A) J''T'' | \left\{ A_{\frac{1}{2}} \left[-\vec{\nabla}_{(A-1)}^2 + \vec{y}_{(A-1)}^2 \right] + A_{\frac{1}{2}} \left[-\vec{\nabla}_{(A)}^2 + \vec{y}_{(A)}^2 \right] \right\} \\
& | ((elj)'_{A-1}, (elj)'_A) J''T'' \rangle \\
& = \sum_{\substack{\overline{(EK\Delta J\bar{I}\bar{I}T)}}_{J''T''}} \sum_{(elj)_{A-1}, (elj)_A} \left[A(e_{A-1} + \frac{3}{2}) + A(e_A + \frac{3}{2}) \right] \\
& \times \langle \overline{(EK\Delta J\bar{I}\bar{I}T)}; ((elj)_{A-1}, (elj)_A) J''T'' || EK\Delta J\bar{I}\bar{I}T \rangle \\
& \times \langle \overline{(EK\Delta J\bar{I}\bar{I}T)}; ((elj)_{A-1}, (elj)_A) J''T'' || EK'\Delta' J\bar{I}\bar{I}T \rangle .
\end{aligned} \tag{9}$$

The expectation value of the two-particle operators from Eq. (7) can be obtained by expressing them into the single-particle form. This may be accomplished by means of Jacobi coordinates

$$\begin{cases} \vec{\xi}_{-1} = \frac{1}{\sqrt{2}}(\vec{y}_{(A-1)} + \vec{y}_{(A)}) \\ \vec{\xi}_2 = \frac{1}{\sqrt{2}}(\vec{y}_{(A-1)} - \vec{y}_{(A)}) . \end{cases} \tag{10}$$

With their use one can rewrite the operator as

$$\frac{1}{2} \left[-(\vec{\nabla}_{(A-1)} - \vec{\nabla}_{(A)})^2 + (\vec{y}_{(A-1)} - \vec{y}_{(A)})^2 \right] = 2 \frac{1}{2} \left(-\vec{\nabla}_{(2)}^2 + \vec{\xi}_{(2)}^2 \right) . \tag{11}$$

The same transformation should be accomplished for the wave functions too. The antisymmetric two-particle oscillator shell model functions may be expanded in terms

of vector coupled products of the $\psi_{elj\pi t m_j m_t}(\xi_2)$ functions depending on the intrinsic Jacobi variable ξ_2 with the $\psi_{(el)_{-1}}(\vec{\xi}_{-1})$ functions depending on the Jacobi coordinate with the nonpositive index $\vec{\xi}_{-1}$ [6]

$$\begin{aligned} |((elj)_{A-1}, (elj)_A)J''T''\rangle &= \sum_{(el)_{-1}, elj\pi t} |((el)_{-1}, elj\pi t)J''T''\rangle \\ &\times \langle ((elj)_{A-1}, (elj)_A)J''T'' | ((el)_{-1}, elj\pi t)J''T'' \rangle. \end{aligned} \quad (12)$$

Here $elj\pi t m_j m_t$ characterize the separated two-particle subsystem, and Jacobi coordinate with nonpositive index is proportional to the centre-of-mass coordinate of this separated two-particle subsystem. Now, the functions $\psi_{elj\pi t m_j m_t}(\xi_2)$ are eigenfunctions of the operator $\frac{1}{2}(-\vec{\nabla}_{(2)}^2 + \vec{\xi}_{(2)}^2)$ as well as orthogonal and normalized, thus for the two-particle term of Eq. (7) one has

$$\begin{aligned} &\langle ((elj)_{A-1}, (elj)_A)J''T'' | (A-1)\frac{1}{2} [-(\vec{\nabla}_{(A-1)} - \vec{\nabla}_{(A)})^2 + (\vec{y}_{(A-1)} - \vec{y}_{(A)})^2] \\ &|((elj)'_{A-1}, (elj)'_A)J''T''\rangle \\ &= \sum_{\substack{(el)_{-1}, elj\pi t \\ (el)'_{-1}, (elj\pi t)'}} \langle ((elj)_{A-1}, (elj)_A)J''T'' | ((el)_{-1}, elj\pi t)J''T''\rangle \\ &\times \langle ((elj)'_{A-1}, (elj)'_A)J''T'' | ((el)'_{-1}, (elj\pi t)')J''T''\rangle \\ &\times 2(A-1) \langle ((el)_{-1}, elj\pi t)J''T'' | \frac{1}{2}(-\vec{\nabla}_{(2)}^2 + \vec{\xi}_{(2)}^2) | ((el)'_{-1}, (elj\pi t)')J''T''\rangle \\ &= \sum_{(el)_{-1}, elj\pi t} 2(A-1) \left(e + \frac{3}{2}\right) \langle ((elj)_{A-1}, (elj)_A)J''T'' | ((el)_{-1}, elj\pi t)J''T''\rangle \\ &\times \langle ((elj)'_{A-1}, (elj)'_A)J''T'' | ((el)_{-1}, elj\pi t)J''T''\rangle. \end{aligned} \quad (13)$$

Here e is the number of oscillator quanta corresponding to the intrinsic motion of the separated two-particle subsystem. It is convenient to extract the coefficient $2(A-1)(e + \frac{3}{2})$ out of the Eq. (13) and move this coefficient to the right-hand side of Eq. (9). After that it obtains the form

$$\begin{aligned}
& \sum_{\overline{(EK\Delta J\pi T)}_{J''T''}} \sum_{(elj)_{A-1}, (elj)_A} [A(e_{A-1} + e_A) + 3] \\
& \times \langle \overline{(EK\Delta J\pi T)}; ((elj)_{A-1}, (elj)_A) J''T'' || EK\Delta J\pi T \rangle \\
& \times \langle \overline{(EK\Delta J\pi T)}; ((elj)_{A-1}, (elj)_A) J''T'' || EK'\Delta' J\pi T \rangle . \tag{14}
\end{aligned}$$

Taking into account Eqs. (13) and (14) the centre-of-mass Hamiltonian matrix element becomes

$$\begin{aligned}
& \langle EK\Delta J\pi T | \frac{H_{c.m.}}{\hbar\omega} | EK'\Delta' J\pi T \rangle \\
& = \sum_{\overline{(EK\Delta J\pi T)}_{J''T''}} \sum_{(elj)_{A-1}, (elj)_A} \langle \overline{(EK\Delta J\pi T)}; ((elj)_{A-1}, (elj)_A) J''T'' || EK\Delta J\pi T \rangle \\
& \times \langle \overline{(EK\Delta J\pi T)}; ((elj)'_{A-1}, (elj)'_A) J''T'' || EK'\Delta' J\pi T \rangle \\
& \times \left\{ \left[\frac{A}{2}(e_{A-1} + e_A) + \frac{3}{2} \right] \delta_{(elj)_{A-1}, (elj)'_{A-1}} \delta_{(elj)_A, (elj)'_A} \right. \\
& - (A-1) \sum_{elj\pi t, (el)_{-1}} e \langle ((elj)_{A-1}, (elj)_A) J''T'' | ((el)_{-1}, elj\pi t) J''T'' \rangle \\
& \left. \times \langle ((elj)'_{A-1}, (elj)'_A) J''T'' | ((el)_{-1}, elj\pi t) J''T'' \rangle \right\} . \tag{15}
\end{aligned}$$

It is possible to express the sum over the single-particle energy in terms of the total energy E . For this, let us express the total Hamiltonian, defined as the sum of single-particle operators, in terms of the sum of two-particle operators

$$\frac{H}{\hbar\omega} = \sum_{i=1}^A \frac{1}{2} \left\{ -\vec{\nabla}_{(i)}^2 + \vec{y}_{(i)}^2 \right\} = \sum_{i=1}^A h_{(i)} = \frac{1}{A-1} \sum_{\substack{i,j=1 \\ i < j}}^A \left(h_{(i)} + h_{(j)} \right) . \tag{16}$$

Now, the total Hamiltonian matrix element between the oscillator shell model functions assumes the form

$$\langle EK\Delta J\pi T | \frac{H}{\hbar\omega} | EK'\Delta' J\pi T \rangle$$

$$\begin{aligned}
&= \frac{1}{A-1} \binom{A}{2} \langle EK \Delta J I I T | (h_{(A-1)} + h_{(A)}) | EK' \Delta' J I I T \rangle \\
&= \sum_{\substack{\overline{\overline{(EK \Delta J I I T)}} \\ J'' T''}} \sum_{(elj)_{A-1}, (elj)_A} \frac{A}{2} (e_{A-1} + \frac{3}{2} + e_A + \frac{3}{2}) \\
&\times \langle \overline{\overline{(EK \Delta J I I T)}}; ((elj)_{A-1}, (elj)_A) J'' T'' || EK \Delta J I I T \rangle \\
&\times \langle \overline{\overline{(EK \Delta J I I T)}}; ((elj)_{A-1}, (elj)_A) J'' T'' || EK' \Delta' J I I T \rangle . \tag{17}
\end{aligned}$$

The oscillator shell model functions are the eigenfunctions of the total Hamiltonian, so the corresponding matrix element is by definition

$$\langle EK \Delta J I I T | \frac{H}{\hbar \omega} | EK' \Delta' J I I T \rangle = \left(E + \frac{3}{2} A \right) \delta_{K \Delta, K' \Delta'} . \tag{18}$$

Comparing Eqs. (17) and (18) one obtains

$$\begin{aligned}
E \cdot \delta_{K \Delta, K' \Delta'} &= \sum_{\substack{\overline{\overline{(EK \Delta J I I T)}} \\ J'' T''}} \sum_{(elj)_{A-1}, (elj)_A} \frac{A}{2} (e_{A-1} + e_A) \\
&\times \langle \overline{\overline{(EK \Delta J I I T)}}; ((elj)_{A-1}, (elj)_A) J'' T'' || EK \Delta J I I T \rangle \\
&\times \langle \overline{\overline{(EK \Delta J I I T)}}; ((elj)_{A-1}, (elj)_A) J'' T'' || EK' \Delta' J I I T \rangle . \tag{19}
\end{aligned}$$

Inserting in the equality (15) the expression (19) and introducing the coefficients of expansion of the oscillator shell model functions in terms of the ones with singled out dependence on the intrinsic coordinates of two last particles (SCFP's), we get the final expression for the centre-of-mass Hamiltonian matrix element

$$\begin{aligned}
&\langle EK \Delta J I I T | \frac{H_{c.m.}}{\hbar \omega} | EK' \Delta' J I I T \rangle = \left(E + \frac{3}{2} \right) \delta_{K \Delta, K' \Delta'} - (A-1) \sum_{elj \pi t} e \\
&\times \sum_{(el)_{-1}, J'' T''} \sum_{\overline{\overline{(EK \Delta J I I T)}}} \langle \overline{\overline{(EK \Delta J I I T)}}; ((el)_{-1}, elj \pi t) J'' T'' || EK \Delta J I I T \rangle \\
&\times \langle \overline{\overline{(EK \Delta J I I T)}}; ((el)_{-1}, elj \pi t) J'' T'' || EK' \Delta' J I I T \rangle . \tag{20}
\end{aligned}$$

3 Eigenvector problem of the real symmetric matrix

For the centre-of-mass Hamiltonian matrix obtained, the general eigenvalue problem should be solved. Usually, this is carried out by some numerical methods. However, numerical methods are inefficient for highly degenerate subspaces, what is the case. The problem at hand is really less complicated, since the eigenvalues of the matrix under consideration are known. Moreover, this problem can be generalized, since eigenvalues of the quadratic Casimir operators for almost all physical interesting groups are well defined. So, the problem can be formulated as to obtain the eigenvectors of the real symmetric matrix \mathbf{M} , when its eigenvalues are known. We present here a new method which allows one to find the analytical expressions of the orthonormalized system of eigenvectors, corresponding to the chosen eigenvalue of the matrix. Since the symmetrical properties of functions are usually formulated in terms of eigenvalues of the quadratic Casimir operators, this method makes it possible to obtain the set of functions with necessary symmetrical characteristics. Let us outline shortly the essence of the method.

It is well-known that the real symmetric matrix \mathbf{M} is expressible as the spectral decomposition of idempotent matrices [8]

$$\mathbf{M} = \sum_{\alpha=1}^s \lambda_{\alpha} \mathbf{P}_{\alpha} . \quad (21)$$

Here $(\mathbf{P}_{\alpha})_{n \times n}$ is the idempotent matrix of the rank r_{α} , and λ_{α} denote the distinct eigenvalues of the matrix \mathbf{M} . Idempotent matrices are known to obey the condition

$$\mathbf{P}_{\alpha} \cdot \mathbf{P}_{\beta} = \delta_{\alpha\beta} \mathbf{P}_{\alpha} . \quad (22)$$

Using the Cayley-Hamilton theorem [8], the matrix \mathbf{P}_{α} may be expressed in terms of the original matrix \mathbf{M} and its eigenvalues

$$\mathbf{P}_\alpha = \prod_{\substack{\beta=1 \\ \beta \neq \alpha}}^s (\lambda_\beta - \mathbf{M}) / \prod_{\substack{\beta=1 \\ \beta \neq \alpha}}^s (\lambda_\beta - \lambda_\alpha) . \quad (23)$$

It is known, that the spectral decomposition of an idempotent matrix is [8]

$$\mathbf{P}_\alpha = \tilde{\mathbf{F}}_\alpha \cdot \tilde{\mathbf{F}}_\alpha^+ , \quad (24)$$

where the matrix $\tilde{\mathbf{F}}_\alpha$ is composed of eigenvectors of the matrix \mathbf{P}_α , corresponding to its eigenvalues equal to one. In the case of real symmetric matrices, $\tilde{\mathbf{F}}_\alpha^+$ is the transposed matrix. The eigenvectors satisfy the usual normalization condition $\tilde{\mathbf{F}}_\alpha^+ \cdot \tilde{\mathbf{F}}_\alpha = 1$.

Taking into account the above spectral decomposition of the matrix \mathbf{P}_α it is straightforward to show that

$$\mathbf{M} \cdot \tilde{\mathbf{F}}_\alpha = \tilde{\mathbf{F}}_\alpha \cdot \lambda_\alpha . \quad (25)$$

It means that columns of the matrix $\tilde{\mathbf{F}}_\alpha$ are eigenvectors of the matrix \mathbf{M} corresponding to the eigenvalue λ_α .

The problem of constructing the eigenvectors for an idempotent matrix was once resolved in the case of coefficients of fractional parentage for antisymmetric wave functions [7]. The method is based on the observation that the spectral decomposition of the matrix \mathbf{P} is defined only within an orthogonal transformation. So, the matrix \mathbf{F} defined by

$$\mathbf{F} = \tilde{\mathbf{F}} \cdot \mathbf{G} , \quad (26)$$

also gives the spectral decomposition of the matrix \mathbf{P} . The orthogonal matrix \mathbf{G} has $r(r-1)/2$ independent parameters, they may be chosen in a way which allows one to fix the corresponding number of the matrix \mathbf{F} elements. The best choice is

$$(F_\alpha)_{ij} = 0 \quad \text{if} \quad 1 \leq i < j \leq r_\alpha . \quad (27)$$

In such a case the expressions for eigenvectors \mathbf{F}_α of the real symmetric matrix \mathbf{M} corresponding to the eigenvalue λ_α take the form

$$\left\{ \begin{array}{l} (F_\alpha)_{11}^2 = (P_\alpha)_{11} , \\ (F_\alpha)_{j1} = \frac{(P_\alpha)_{1j}}{(F_\alpha)_{11}} , \\ (F_\alpha)_{ii}^2 = (P_\alpha)_{ii} - \sum_{k=1}^{i-1} (F_\alpha)_{ik}^2 , \\ (F_\alpha)_{ji} = \frac{1}{(F_\alpha)_{ii}} \left\{ (P_\alpha)_{ij} - \sum_{k=1}^{i-1} (F_\alpha)_{ik} (F_\alpha)_{jk} \right\} , \end{array} \right. \quad (28)$$

for every value of $i = 2, 3, \dots, r$ and the corresponding set of $j = i + 1, i + 2, \dots, n$. Here n is the dimension and r is the rank of the matrix \mathbf{P}_α . The r columns of the matrix \mathbf{F}_α may be obtained from the set of any r linearly independent rows of the matrix \mathbf{P}_α . Consider the first nonzero row of the matrix \mathbf{P}_α . Let it be the k -th. Then the calculation begins with this k -th row of the matrix \mathbf{P}_α : $(F_\alpha)_{11}^2 = (P_\alpha)_{kk}$ and $(F_\alpha)_{j1} = \frac{(P_\alpha)_{kj}}{(F_\alpha)_{11}}$. Because the matrix \mathbf{P}_α is symmetric, every next i -th row calculation starts from the diagonal element $(F_\alpha)_{ii}$. This i -th row of the matrix \mathbf{P}_α will be linearly independent of rows just calculated if the obtained diagonal element $(F_\alpha)_{ii} \neq 0$. In this case the corresponding column of the matrix \mathbf{F}_α could be obtained. If this condition is not fulfilled, the next row of the matrix \mathbf{P}_α should be tested. Since the matrix \mathbf{P}_α has rank r , it will surely contain r linearly independent rows. Positive values of F_{ii} are convenient, because the overall sign of the eigenvector is arbitrary.

The outlined method for constructing the eigenvectors with necessary characteristics can be applied for diagonalization of the $H_{c.m.}$ matrix. The obtained eigenvectors \mathbf{F} will be the sets of coefficients $a_{K\Delta;00,\Gamma}^{EJIT}$ introduced in Ref. [6]. Then the coefficients of expansion of the oscillator shell model functions in terms of A -particle oscillator functions with singled out dependence on intrinsic coordinates of two last particles and with eliminated spurious states (CESO's), as well as the intrinsic density matrices can be calculated.

4 Numerical benchmarks

Let us use as a benchmark the formation of the matrix $\langle H_{\text{c.m.}} \rangle$, its diagonalization, and calculation of the CESO's and intrinsic density matrices for the $A = 3$ case.

The eigenvalues of the $H_{\text{c.m.}}$ matrix makes the sequence $\frac{3}{2}, \frac{5}{2} \dots E - E_{\min} + \frac{3}{2}$, where E is the total oscillator energy value of the state.

Let us start from the case $E = E_{\min}$. Then, for three particles there is only one configuration K_1 , and the total angular momentum as well as the isospin can take only one value

$$K_1 \equiv \left(00\frac{1}{2}\right)^3: \quad J\pi T = \frac{1}{2}^+ \frac{1}{2}.$$

The state (1) in the configuration space will be denoted by the sequence of single shells $(elj)^n$ accompanied with JT values in correspondence with the momentum coupling scheme.

$$(1) \quad \left(\left(00\frac{1}{2}\right)^3\right)_{\frac{1}{2}\frac{1}{2}}.$$

In the case when $E = E_{\min}$, the J and T could take only the displayed values. For brevity, let us deal below with the same $J\pi T = \frac{1}{2}^+ \frac{1}{2}$ values also for higher total oscillator energies.

In the case of three particles and $E = E_{\min} = 0$ the matrix of the operator $H_{\text{c.m.}}$ is the single number

$$\left\langle \frac{H_{\text{c.m.}}}{\hbar\omega} \right\rangle_{A=3, E=0, J\pi T=\frac{1}{2}^+ \frac{1}{2}} = \frac{3}{2}.$$

This is why the oscillator shell model states characterized by E_{\min} contain the centre of mass in its ground state and all $a_{K\Delta;00,\Gamma}^{EJ\pi T} = 1$.

Now the centre of mass is not excited, hence the CESO's coincide with the coefficients of expansion of the oscillator shell model function in terms of the ones with singled out dependence on intrinsic coordinates of two last particles. They are presented in Table 1. Here the more usual spectroscopic notation $^{2s+1}l_j$ is used to denote the state of a subsystem containing two separated particles.

The intrinsic density matrices, in the case when $A = 3$, $E = 0$, $JT = \frac{1}{2}\frac{1}{2}$ are

$$\mathbf{W}(^1S_0) = \left(\begin{array}{c} \frac{1}{2} \end{array} \right) , \quad \mathbf{W}(^3S_1) = \left(\begin{array}{c} \frac{1}{2} \end{array} \right) .$$

In the case of $E = E_{\min} + 1 = 1$, there are two configurations and a number of total angular momentum and isospin values appear:

$$\begin{aligned} K_1 &\equiv \left(00\frac{1}{2}\right)^2 \left(11\frac{1}{2}\right): & J\pi T &= \frac{1}{2}^-\frac{1}{2}, \frac{1}{2}^-\frac{3}{2}, \frac{3}{2}^-\frac{1}{2}, \\ K_2 &\equiv \left(00\frac{1}{2}\right)^2 \left(11\frac{3}{2}\right): & J\pi T &= \frac{1}{2}^-\frac{1}{2}, \frac{3}{2}^-\frac{1}{2}, \frac{3}{2}^-\frac{3}{2}, \frac{5}{2}^-\frac{1}{2}. \end{aligned}$$

Now, the $J\pi T = \frac{1}{2}^+\frac{1}{2}$ values may be obtained for the three states in the configuration space:

- (1) $\left(\left(00\frac{1}{2}\right)^2 01, \left(11\frac{1}{2}\right)\frac{1}{2}\frac{1}{2}\right)\frac{1}{2}\frac{1}{2},$
- (2) $\left(\left(00\frac{1}{2}\right)^2 10, \left(11\frac{1}{2}\right)\frac{1}{2}\frac{1}{2}\right)\frac{1}{2}\frac{1}{2},$
- (3) $\left(\left(00\frac{1}{2}\right)^2 10, \left(11\frac{3}{2}\right)\frac{3}{2}\frac{1}{2}\right)\frac{1}{2}\frac{1}{2}.$

In order to calculate the $\langle H_{\text{c.m.}} \rangle$ matrix, the coefficients of expansion of the oscillator shell model functions in terms of the ones with singled out dependence on intrinsic coordinates of two last particles for $E = E_{\min} + 1$ should be known. They are presented in Table 2. Here only the values of the indices different from the preceding row are shown.

Since there are three states in the configuration space, the dimension of the matrix of the operator $H_{\text{c.m.}}$ will be three:

$$\left\langle \frac{H_{\text{c.m.}}}{\hbar\omega} \right\rangle_{A=3, E=1, J\pi T=\frac{1}{2}^-\frac{1}{2}} = \begin{pmatrix} 2 & \frac{1}{6} & \frac{2}{3\sqrt{2}} \\ & \frac{14}{9} & \frac{2}{9\sqrt{2}} \\ & & \frac{35}{18} \end{pmatrix},$$

where the rows and columns of the matrix $\langle H_{\text{c.m.}} \rangle$ are labeled by the states in the configuration space that are taken in the sequence (1), (2), (3). Due to the symmetric nature of this matrix here we show only its upper triangle. The rank of the idempotent matrices of this matrix is two, so there will be only two normalized eigenvectors. After

diagonalization of the matrix $\langle H_{\text{c.m.}} \rangle$ according to the outlined above method and using Eq. (28), one may obtain

$$\langle a_{K\Delta;00,\Gamma} \rangle^{A=3,E=1,JIT=\frac{1}{2}-\frac{1}{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{3\sqrt{2}} & \frac{4}{3\sqrt{2}} \\ -\frac{2}{3} & -\frac{1}{3} \end{pmatrix},$$

where rows of the matrix $\langle a_{K\Delta;00,\Gamma} \rangle$ are labeled by the states in the configuration space that are taken in the same sequence (1), (2), (3) as for $\langle H_{\text{c.m.}} \rangle$, columns are labeled by the additional integer quantum number Γ which can take two values: 1 and 2, in correspondence with the rank of the matrix $\langle H_{\text{c.m.}} \rangle$ or, in another words, with the number of its linearly independent eigenvectors. It should be stressed, that the empty place instead of the first element of the second column means that its value is equal to zero, in full accordance with prescriptions of the method.

The results of CESO's calculations are presented in Table 3. It should be stressed, that CESO's are labeled with the set of indices: $\overline{\overline{(EK\Delta JIIT)}}((el)_{-1}, elj\pi t)J''T''$ and the additional integer quantum number Γ . There are only different sets of them, but they can belong to all states in the configuration space, regardless of the origin of the ground configuration. For example, if seven different sets of CESO's indices are taken from the state (1), then only one can be taken from the state (2), since the remaining six have the same sets of indices as in the state (1). At last, a set of indices will be taken from the state (3), since the remaining four may be met in states (1) and (3). Table 3 shows only the values of the indices different from the preceding row, whereas CESO's which are absent here have zero values.

Finally, by means of obtained CESO's the intrinsic density matrices can be calculated.

The intrinsic density matrices in the case of $A = 3$, $E = 1$, $JT = \frac{1}{2}\frac{1}{2}$:

$$\mathbf{W}({}^1S_0) = \begin{pmatrix} \frac{1}{4} \end{pmatrix}, \quad \mathbf{W}({}^3S_1) = \begin{pmatrix} \frac{1}{4} & 0 \\ & \frac{1}{2} \end{pmatrix},$$

$$\mathbf{W}({}^3P_0) = \begin{pmatrix} \frac{1}{12} & -\frac{1}{6} \\ & \frac{1}{3} \end{pmatrix}, \quad \mathbf{W}({}^1P_1) = \begin{pmatrix} \frac{1}{4} \end{pmatrix}, \quad \mathbf{W}({}^3P_1) = \begin{pmatrix} \frac{1}{6} & \frac{1}{6} \\ & \frac{1}{6} \end{pmatrix}.$$

The rows and columns of the intrinsic density matrices are labeled by the additional integer quantum number Γ , preserving ascending order, so the dimensions of the matrices can vary from 1 to 2. Only the upper triangle of the intrinsic density matrices is shown due to its symmetry under index permutation.

In the case of $E = E_{\min} + 2 = 2$ there are six configurations that produce a number of total angular momentum and isospin values:

$$\begin{aligned} K_1 &\equiv \left(00\frac{1}{2}\right)^2 \left(20\frac{1}{2}\right): & J\pi T &= \frac{1}{2}^+ \frac{1}{2}, \frac{1}{2}^+ \frac{3}{2}, \frac{3}{2}^+ \frac{1}{2}, \\ K_2 &\equiv \left(00\frac{1}{2}\right)^2 \left(22\frac{3}{2}\right): & J\pi T &= \frac{1}{2}^+ \frac{1}{2}, \frac{3}{2}^+ \frac{1}{2}, \frac{3}{2}^+ \frac{3}{2}, \frac{5}{2}^+ \frac{1}{2}, \\ K_3 &\equiv \left(00\frac{1}{2}\right)^2 \left(22\frac{5}{2}\right): & J\pi T &= \frac{3}{2}^+ \frac{1}{2}, \frac{5}{2}^+ \frac{1}{2}, \frac{5}{2}^+ \frac{3}{2}, \frac{7}{2}^+ \frac{1}{2}, \\ K_4 &\equiv \left(00\frac{1}{2}\right)^2 \left(11\frac{1}{2}\right)^2: & J\pi T &= \frac{1}{2}^+ \frac{1}{2}, \frac{1}{2}^+ \frac{3}{2}, \frac{3}{2}^+ \frac{1}{2}, \\ K_5 &\equiv \left(00\frac{1}{2}\right) \left(11\frac{1}{2}\right) \left(11\frac{3}{2}\right): & J\pi T &= \frac{1}{2}^+ \frac{1}{2}, \frac{1}{2}^+ \frac{3}{2}, \frac{3}{2}^+ \frac{1}{2}, \frac{3}{2}^+ \frac{3}{2}, \frac{5}{2}^+ \frac{1}{2}, \frac{5}{2}^+ \frac{3}{2}, \\ K_6 &\equiv \left(00\frac{1}{2}\right)^2 \left(11\frac{1}{2}\right)^2: & J\pi T &= \frac{1}{2}^+ \frac{1}{2}, \frac{1}{2}^+ \frac{3}{2}, \frac{3}{2}^+ \frac{1}{2}, \frac{3}{2}^+ \frac{3}{2}, \frac{5}{2}^+ \frac{1}{2}, \frac{5}{2}^+ \frac{3}{2}, \frac{7}{2}^+ \frac{1}{2}. \end{aligned}$$

Here and bellow the remarks, about notations mentioned above, will be valid, so the data will be presented in more concise manner.

The $J\pi T = \frac{1}{2}^+ \frac{1}{2}$ values could be obtained for nine states in the configuration space:

- (1) $\left(\left(00\frac{1}{2}\right)^2 01, \left(20\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}\right) \frac{1}{2}\frac{1}{2},$
- (2) $\left(\left(00\frac{1}{2}\right)^2 10, \left(20\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}\right) \frac{1}{2}\frac{1}{2},$
- (3) $\left(\left(00\frac{1}{2}\right)^2 10, \left(22\frac{3}{2}\right) \frac{3}{2}\frac{1}{2}\right) \frac{1}{2}\frac{1}{2},$
- (4) $\left(\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}, \left(11\frac{1}{2}\right)^2 01\right) \frac{1}{2}\frac{1}{2},$
- (5) $\left(\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}, \left(11\frac{1}{2}\right)^2 10\right) \frac{1}{2}\frac{1}{2},$
- (6) $\left(\left(\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}, \left(11\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}\right) 10, \left(11\frac{3}{2}\right) \frac{3}{2}\frac{1}{2}\right) \frac{1}{2}\frac{1}{2},$
- (7) $\left(\left(\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}, \left(11\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}\right) 11, \left(11\frac{3}{2}\right) \frac{3}{2}\frac{1}{2}\right) \frac{1}{2}\frac{1}{2},$
- (8) $\left(\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}, \left(11\frac{3}{2}\right)^2 01\right) \frac{1}{2}\frac{1}{2},$
- (9) $\left(\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}, \left(11\frac{3}{2}\right)^2 10\right) \frac{1}{2}\frac{1}{2}.$

The number of the coefficients of expansion of the oscillator shell model functions in terms of the ones with singled out dependence on the intrinsic coordinates of two last particles is 92, so they will not be presented here for the reasons of space.

Since there are nine states in the configuration space, the dimension of the matrix of the operator $H_{\text{c.m.}}$ will be nine

$$\left\langle \frac{H_{\text{c.m.}}}{\hbar\omega} \right\rangle_{A=3, E=2}^{J\pi T=\frac{1}{2}^+\frac{1}{2}} = \begin{pmatrix} \frac{13}{6} & 0 & 0 & -\frac{1}{3\sqrt{6}} & -\frac{1}{3\sqrt{6}} & -\frac{2}{3\sqrt{6}} & \frac{2}{3\sqrt{2}} & -\frac{1}{3\sqrt{3}} & \frac{5}{3\sqrt{15}} \\ & \frac{13}{6} & 0 & \frac{1}{\sqrt{6}} & \frac{1}{9\sqrt{6}} & \frac{2}{9\sqrt{6}} & -\frac{2}{9\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{5}{9\sqrt{15}} \\ & & \frac{13}{6} & 0 & \frac{20}{9\sqrt{30}} & -\frac{5}{9\sqrt{30}} & \frac{5}{9\sqrt{10}} & 0 & -\frac{2}{9\sqrt{3}} \\ & & & \frac{7}{3} & \frac{1}{6} & 0 & -\frac{2}{3\sqrt{3}} & 0 & 0 \\ & & & & \frac{17}{9} & \frac{2}{9} & 0 & 0 & 0 \\ & & & & & \frac{7}{3} & -\frac{1}{6\sqrt{3}} & \frac{1}{3\sqrt{2}} & \frac{5}{9\sqrt{10}} \\ & & & & & & 2 & -\frac{1}{3\sqrt{6}} & \frac{5}{3\sqrt{30}} \\ & & & & & & & \frac{7}{3} & -\frac{5}{6\sqrt{5}} \\ & & & & & & & & \frac{19}{9} \end{pmatrix}$$

The rank of the idempotent matrices of this matrix is 4. The diagonalization of the matrix $\langle H_{\text{c.m.}} \rangle$ is accomplished within the framework of the outlined above method using

$$\langle a_{K\Delta;00,\Gamma} \rangle_{A=3, E=2}^{J\pi T=\frac{1}{2}^+\frac{1}{2}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & 0 \\ -\frac{1}{3\sqrt{2}} & \frac{2}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & -\frac{1}{\sqrt{6}} & 0 & \frac{1}{3} \\ \frac{2}{9\sqrt{3}} & \frac{1}{9\sqrt{6}} & -\frac{2}{9\sqrt{10}} & -\frac{1}{3} \\ \frac{4}{9\sqrt{3}} & \frac{2}{9\sqrt{6}} & \frac{5}{9\sqrt{10}} & \frac{1}{3} \\ -\frac{4}{9} & -\frac{2}{9\sqrt{2}} & -\frac{5}{3\sqrt{30}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{1}{\sqrt{3}} & 0 & -\frac{1}{3\sqrt{2}} \\ -\frac{20}{9\sqrt{30}} & -\frac{5}{9\sqrt{15}} & \frac{2}{9} & -\frac{5}{3\sqrt{10}} \end{pmatrix}$$

The corresponding CESO's are presented in Table 4.

The intrinsic density matrices in the case of $A = 3$, $E = 2$, $JT = \frac{1}{2}\frac{1}{2}$ are:

$$\begin{aligned}
\mathbf{W}({}^1S_0) &= \begin{pmatrix} \frac{3}{8} \end{pmatrix}, \quad \mathbf{W}({}^1S'_0) = \begin{pmatrix} \frac{1}{24} & -\frac{1}{6\sqrt{2}} \\ & \frac{1}{3} \end{pmatrix}, \quad \mathbf{W}({}^3D_1) = \begin{pmatrix} \frac{1}{4} \end{pmatrix}, \\
\mathbf{W}({}^3S_1) &= \begin{pmatrix} \frac{1}{24} & -\frac{1}{6\sqrt{2}} & 0 \\ & \frac{1}{3} & 0 \\ & & \frac{1}{4} \end{pmatrix}, \quad \mathbf{W}({}^3P_2) = \begin{pmatrix} \frac{5}{108} & \frac{5}{54\sqrt{2}} & -\frac{5}{36\sqrt{30}} & \frac{5}{36\sqrt{3}} \\ & \frac{5}{54} & -\frac{5}{36\sqrt{15}} & \frac{5}{18\sqrt{6}} \\ & & \frac{1}{72} & -\frac{5}{36\sqrt{10}} \\ & & & \frac{5}{36} \end{pmatrix}, \\
\mathbf{W}({}^3P_0) &= \begin{pmatrix} \frac{1}{108} & \frac{1}{54\sqrt{2}} & -\frac{5}{18\sqrt{30}} & \frac{1}{18\sqrt{3}} \\ & \frac{1}{54} & -\frac{5}{18\sqrt{15}} & -\frac{1}{9\sqrt{6}} \\ & & \frac{5}{18} & \frac{5}{9\sqrt{10}} \\ & & & \frac{1}{9} \end{pmatrix}, \quad \mathbf{W}({}^1P_1) = \begin{pmatrix} \frac{1}{12} & \frac{1}{6\sqrt{2}} & 0 \\ & \frac{1}{6} & 0 \\ & & \frac{1}{2} \end{pmatrix}, \\
\mathbf{W}({}^3P_1) &= \begin{pmatrix} \frac{1}{36} & \frac{1}{18\sqrt{2}} & \frac{5}{12\sqrt{30}} & -\frac{1}{12\sqrt{3}} \\ & \frac{1}{18} & \frac{5}{12\sqrt{15}} & -\frac{1}{6\sqrt{6}} \\ & & \frac{5}{24} & -\frac{5}{12\sqrt{10}} \\ & & & \frac{1}{4} \end{pmatrix}, \quad \mathbf{W}({}^3S'_1) = \begin{pmatrix} \frac{3}{8} \end{pmatrix}.
\end{aligned}$$

Here the primes differentiate the notations of the states with coinciding orbital momenta.

So far, for all excitation energies under consideration, only the subspaces corresponding to the nonspurious motion were extracted. It should be mentioned, that the same procedure can be carried out also for other subspaces corresponding to the possible excitements of the centre of mass. For this, only the value of excitation energy of the centre of mass (the one from the sequence $\frac{3}{2}, \frac{5}{2} \dots E - E_{\min} + \frac{3}{2}$) should be changed.

5 Conclusions

The proposed new method for elimination of 'spurious' states enables one to produce two-particle oscillator intrinsic density matrices without exploiting the ICFP's which untill now are not available for an arbitrary number of oscillator quanta. The distinct advantage of this method is in the complete rejection of group-theoretical classification of antisymmetric and translationally invariant many-particle states. So, a benefit could be gained due to simplicity and comprehensibility of such kind calculations. Another distinct feature of the method is that it do not involve any numerical diagonalization and orthogonalization, i.e. it is stable numerically. So, the scale of calculations is determined only by available computation capabilities. Software tools based on a root rational fraction expression of numbers for calculation of CESO's and two-particle oscillator intrinsic density matrices were developed and implemented in a computer code. The theoretical formulation have been illustrated by calculation of two-particle oscillator intrinsic density matrices in the case of $A = 3$, $E = E_{\min}, E_{\min} + 1, E_{\min} + 2$, and $JIT = \frac{1}{2} + \frac{1}{2}$.

The presented procedure for projecting off the required oscillator shell model subspace can be generalized by proposing a new method for calculation of analytical expressions of eigenvectors if the analytical expressions of eigenvalues and entries of the real symmetric matrix under consideration are known. Since it is the case for all quadratic Casimir operators, this method can be applied to produce large irreducible subspaces with desired transformational properties.

The new method for elimination of 'spurious' states proposed in this paper enables one to formulate the problem of calculation of two-particle oscillator intrinsic density matrices in such form where this problem is no more complicated as matrix algebra and hence can be solved in a simple and efficient way.

References

- [1] **M. Kretzschmar**, Gruppentheoretische untersuchungen zum schalen modell. I. Die Matchematische theorie des Hamilton-operators," Z. f. Phys., V. B157, p. 433–456 (1960).
- [2] **M. Kretzschmar**, "Gruppentheoretische untersuchungen zum schalen modell. II. Zum problem der translation invarianz," Z. f. Phys., V. B158, p. 284–303 (1960).
- [3] **V. Vanagas**, *Algebraic Methods in Nuclear Theory* (Vilnius: Mokslas, 1971, in russian).
- [4] **I.V. Kurdyumov, Yu.F. Smirnov, K.V. Shitikova and S.Kh.El. Samarai**, "Translationally invariant shell model," Nucl. Phys., V. A145, p. 593-612 (1970).
- [5] **K. Wildermuth, Y.C. Tang**, *A unified theory of the nucleus* (Braunschweig: Vieweg, 1977).
- [6] **A. Deveikis and G. Kamuntavičius**, "Intrinsic density matrices of the nuclear shell model," Lithuanian Phys. J., V. 36, No. 2, p. 83-95 (1996).
- [7] **A. Deveikis and G. Kamuntavičius**, "Coefficients of fraction parentage for nuclear shell model," Lithuanian Phys. J., V. 35, No. 1, p. 14–19 (1995).
- [8] **P. Lankaster**, *Theory of matrices* (New York - London: Academic Press, 1969).

Table 1.

The coefficients of expansion of the oscillator shell model functions in terms of the A -particle oscillator functions with singled out dependence on the intrinsic coordinates of two last particles and with eliminated spurious states for the three-nucleon system: $E = 0$, $K = \left(00\frac{1}{2}\right)^3 \frac{1}{2}\frac{1}{2}$, $\Delta = 1$, $JIT = \frac{1}{2}^+ \frac{1}{2}$.

$\overline{K\Delta}$	\overline{JT}	$(el)_{-1}$	$elsjt$	state	$J''T''$	CESO's
$\left(00\frac{1}{2}\right)^3 \frac{1}{2}\frac{1}{2}$	$\frac{1}{2}\frac{1}{2}$	00	00001	1S_0	01	$-\frac{1}{\sqrt{2}}$
		00	00110	3S_1	10	$\frac{1}{\sqrt{2}}$

Table 2.

The coefficients of expansion of the oscillator shell model functions in terms of the ones with singled out dependence on the intrinsic coordinates of two last particles for the three-nucleon system: $E = 1$, $K = \left(00\frac{1}{2}\right)^2 01 \left(11\frac{1}{2}\right)^1 \frac{1}{2}\frac{1}{2}$, $\Delta = 1$, $JIT = \frac{1}{2}^- \frac{1}{2}$.

$\overline{K\Delta}$	\overline{JT}	$(el)_{-1}$	$elsjt$	state	$J''T''$	SCFP's
$\left(11\frac{1}{2}\right)^1 \frac{1}{2}\frac{1}{2}$	$\frac{1}{2}\frac{1}{2}$	00	00001	1S_0	01	$-\frac{1}{\sqrt{3}}$
$\left(00\frac{1}{2}\right)^2 \frac{1}{2}\frac{1}{2}$		11	00110	3S_1	00	$\frac{1}{2\sqrt{2}}$
		00	11101	3P_0	01	$-\frac{1}{2\sqrt{6}}$
			11010	1P_1	10	$-\frac{1}{2\sqrt{2}}$
		11	00110	3S_1		$\frac{1}{2}$
		00	11111	3P_1	11	$-\frac{1}{2\sqrt{3}}$
		11	00001	1S_0		$\frac{1}{2\sqrt{6}}$

Table 2 (continued).

The coefficients of expansion of the oscillator shell model functions in terms of the ones with singled out dependence on the intrinsic coordinates of two last particles for the three-nucleon system: $E = 1$, $K = \left(00\frac{1}{2}\right)^2 10 \left(11\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}$, $\Delta = 1$, $JIT = \frac{1}{2}^- \frac{1}{2}$.

$\overline{K\Delta}$	\overline{JT}	$(el)_{-1}$	$elsjt$	state	$J''T''$	SCFP's
$\left(11\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}$	$\frac{1}{2}\frac{1}{2}$	00	00110	3S_1	10	$-\frac{1}{\sqrt{3}}$
$\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}$		11			00	$\frac{1}{2\sqrt{2}}$
		00	11101	3P_0	01	$\frac{3}{2\sqrt{6}}$
			11010	1P_1	10	$\frac{1}{6\sqrt{2}}$
		11	00110	3S_1		$-\frac{1}{6}$
		00	11111	3P_1	11	$-\frac{1}{2\sqrt{3}}$
		11	00001	1S_0		$\frac{1}{2\sqrt{6}}$

Table 2 (continued).

The coefficients of expansion of the oscillator shell model functions in terms of the ones with singled out dependence on the intrinsic coordinates of two last particles for the three-nucleon system: $E = 1$, $K = \left(00\frac{1}{2}\right)^2 10 \left(11\frac{3}{2}\right) \frac{3}{2}\frac{1}{2}$, $\Delta = 1$, $JIT = \frac{1}{2}^- \frac{1}{2}$.

$\overline{K\Delta}$	\overline{JT}	$(el)_{-1}$	$elsjt$	state	$J''T''$	SCFP's
$\left(11\frac{3}{2}\right) \frac{3}{2}\frac{1}{2}$	$\frac{3}{2}\frac{1}{2}$	00	00110	3S_1	10	$\frac{1}{\sqrt{3}}$
$\left(00\frac{1}{2}\right) \frac{1}{2}\frac{1}{2}$	$\frac{1}{2}\frac{1}{2}$		11010	1P_1		$\frac{1}{3}$
		11	00110	3S_1		$\frac{1}{3\sqrt{2}}$
		00	11111	3P_1	11	$\frac{1}{\sqrt{6}}$
		11	00001	1S_0		$\frac{1}{\sqrt{3}}$

Table 3.

The coefficients of expansion of the oscillator shell model functions in terms of A -particle oscillator functions with singled out dependence on the intrinsic coordinates of two last particles and with eliminated spurious states for the three-nucleon system : $A = 3, E = 1, JT = \frac{1}{2}\frac{1}{2}$.

$\overline{K\Delta}$	\overline{JT}	$(el)_{-1}$	$elsjt$	state	$J''T''$	CESO's	
						$\Gamma = 1$	$\Gamma = 2$
$\left(00\frac{1}{2}\right)\frac{1}{2}\frac{1}{2}$	$\frac{1}{2}\frac{1}{2}$	11	00001	1S_0	11	$-\frac{1}{2\sqrt{3}}$	0
			00110	3S_1	00	$\frac{1}{6}$	$\frac{1}{3}$
					10	$\frac{1}{3\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$
		00	11101	3P_0	01	$-\frac{1}{2\sqrt{3}}$	$\frac{1}{\sqrt{3}}$
			11010	1P_1	10	$-\frac{1}{2}$	0
			11111	3P_1	11	$-\frac{1}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$
$\left(11\frac{1}{2}\right)\frac{1}{2}\frac{1}{2}$			00001	1S_0	01	$-\frac{1}{\sqrt{6}}$	0
			00110	3S_1	10	$\frac{1}{3\sqrt{6}}$	$-\frac{4}{3\sqrt{6}}$
$\left(11\frac{3}{2}\right)\frac{3}{2}\frac{1}{2}$	$\frac{3}{2}\frac{1}{2}$					$-\frac{2}{3\sqrt{3}}$	$-\frac{1}{3\sqrt{3}}$

Table 4.

The coefficients of expansion of the oscillator shell model functions in terms of A -particle oscillator functions with singled out dependence on the intrinsic coordinates of two last particles and with eliminated spurious states for the three-nucleon system : $A = 3, E = 2, JT = \frac{1}{2}\frac{1}{2}$.

$\overline{K\Delta}$	\overline{JT}	$(el)_{-1}$	$elsjt$	state	$J''T''$	CESO's			
						$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 3$	$\Gamma = 4$
$(00\frac{1}{2})\frac{1}{2}\frac{1}{2}$	$\frac{1}{2}\frac{1}{2}$	20	00001	1S_0	01	$-\frac{1}{2\sqrt{6}}$	0	0	0
		00	20001			$-\frac{1}{2\sqrt{6}}$	$\frac{1}{\sqrt{3}}$	0	0
		20	00110	3S_1	10	$\frac{1}{6\sqrt{6}}$	$-\frac{1}{3\sqrt{3}}$	0	0
		22				0	0	$-\frac{1}{6}$	0
		00	20110			$\frac{3}{2\sqrt{6}}$	0	0	0
		11	11101	3P_0	11	$-\frac{1}{18}$	$-\frac{1}{9\sqrt{2}}$	$\frac{5}{3\sqrt{30}}$	$\frac{1}{3\sqrt{3}}$
			11010	1P_1	00	$\frac{1}{6}$	$\frac{1}{3\sqrt{2}}$	0	0
					10	0	0	0	$\frac{1}{\sqrt{6}}$
			11111	3P_1	01	0	0	0	$-\frac{1}{3\sqrt{2}}$
					11	$\frac{1}{6\sqrt{3}}$	$\frac{1}{3\sqrt{6}}$	$\frac{5}{6\sqrt{10}}$	$-\frac{1}{6}$
			11121	3P_2		$-\frac{5}{18\sqrt{5}}$	$-\frac{5}{9\sqrt{10}}$	$\frac{1}{6\sqrt{6}}$	$-\frac{5}{6\sqrt{15}}$
		00	22110	3D_1	10	0	0	$\frac{1}{2}$	0
$(11\frac{1}{2})\frac{1}{2}\frac{1}{2}$		11	00001	1S_0	11	$\frac{1}{3\sqrt{2}}$	0	0	0
			00110	3S_1	00	$\frac{1}{9\sqrt{6}}$	$-\frac{2}{9\sqrt{3}}$	$-\frac{5}{9\sqrt{5}}$	0
					10	$-\frac{1}{9\sqrt{3}}$	$\frac{4}{9\sqrt{6}}$	$-\frac{5}{9\sqrt{10}}$	0
		00	11101	3P_0	01	$-\frac{1}{9\sqrt{2}}$	$-\frac{1}{9}$	$\frac{5}{3\sqrt{15}}$	$\frac{2}{3\sqrt{6}}$
			11010	1P_1	10	$-\frac{1}{3\sqrt{6}}$	$-\frac{1}{3\sqrt{3}}$	0	$\frac{2}{3\sqrt{2}}$
			11111	3P_1	11	$\frac{1}{9}$	$\frac{2}{9\sqrt{2}}$	$\frac{5}{3\sqrt{30}}$	0

Table 4 (continued).

The coefficients of expansion of the oscillator shell model functions in terms of A -particle oscillator functions with singled out dependence on the intrinsic coordinates of two last particles and with eliminated spurious states for the three-nucleon system : $A = 3, E = 2, JT = \frac{1}{2}\frac{1}{2}$.

$\overline{K\Delta}$	\overline{JT}	$(el)_{-1}$	$elsjt$	state	$J''T''$	CESO's			
						$\Gamma = 1$	$\Gamma = 2$	$\Gamma = 3$	$\Gamma = 4$
$\left(11\frac{3}{2}\right)\frac{3}{2}\frac{1}{2}$	$\frac{3}{2}\frac{1}{2}$	11	00001	1S_0	11	$\frac{1}{3}$	0	0	0
			00110	3S_1	10	$\frac{1}{9\sqrt{6}}$	$-\frac{2}{9\sqrt{3}}$	$\frac{5}{18\sqrt{5}}$	0
					20	$-\frac{5}{9\sqrt{30}}$	$\frac{10}{9\sqrt{15}}$	$\frac{1}{18}$	0
		00	11010	1P_1	10	$-\frac{1}{3\sqrt{3}}$	$-\frac{2}{3\sqrt{6}}$	0	$-\frac{1}{3}$
			11111	3P_1	11	$-\frac{1}{9\sqrt{2}}$	$-\frac{1}{9}$	$-\frac{5}{6\sqrt{15}}$	$\frac{1}{\sqrt{6}}$
			11121	3P_2	21	$\frac{5}{9\sqrt{10}}$	$\frac{5}{9\sqrt{5}}$	$-\frac{1}{6\sqrt{3}}$	$\frac{5}{3\sqrt{30}}$
$\left(20\frac{1}{2}\right)\frac{1}{2}\frac{1}{2}$	$\frac{1}{2}\frac{1}{2}$		00001	1S_0	01	$-\frac{1}{\sqrt{6}}$	0	0	0
			00110	3S_1	10	$\frac{1}{3\sqrt{6}}$	$-\frac{2}{3\sqrt{3}}$	0	0
$\left(22\frac{3}{2}\right)\frac{3}{2}\frac{1}{2}$	$\frac{3}{2}\frac{1}{2}$					0	0	$\frac{1}{3}$	0